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# An Illocutionary Logical Explanation of the Liar Paradox

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This paper uses the resources of illocutionary logic to provide a new understanding of the Liar Paradox. In the system of illocutionary logic of the paper, denials are irreducible counterparts of assertions; denial does not in every case amount to the same as the assertion of the negation of the statement that is denied. Both a Liar statement, (a) Statement (a) is not true, and the statement which it negates can correctly be denied; neither can correctly be asserted. A Liar statement, more precisely, an *attempted* Liar statement, fails to fulfill conditions essential to statements, but no linguistic rules are violated by the attempt. Ordinary language, our ordinary practice of using language, is not inconsistent or incoherent because of the Liar. We are committed to deny Liars, but not to accept or assert them. This understanding of the Liar Paradox and its sources cannot be fully accommodated in a conventional logical system, which fails to mark the distinction between sentences/statements and illocutionary acts of accepting, rejecting, and supposing statements.

## 1. Illocutionary propositional logic

In *Kearns* (*1997*), I developed a system of illocutionary propositional logic. In *Kearns* (*2000a*, *2000b*), I further developed this system and used it to explain (but not 'explain away') the surprise execution paradox. In the present paper, I will use the resources of illocutionary logic to understand and characterize the Liar Paradox, showing both what features of language give rise to the Paradox and how these features are harmless. Speaking our native tongue does not commit us to either assert or accept contradictory statements.

Daniel Vanderveken and John Searle introduced the field of illocutionary logic in *Vanderveken and Searle* (*1985*) and *Vanderveken* (*1990*). However, they think of illocutionary logic in a different way than I do, regarding illocutionary logic as a supplement, or appendix, to standard logic; they focus on very general principles/laws which characterize illocutionary acts of all kinds. I understand illocutionary logic to be a comprehensive subject matter that includes standard logic as a proper part. I develop systems which deal with specific kinds of illocutionary acts, and favour a multiplicity of different systems for 'capturing' the different kinds of illocutionary acts.

I take the fundamental 'linguistic reality' to be constituted by speech acts, or *language acts*. A speech act is a meaningful act performed by using an expression. A speech act or language act can be performed by speaking, writing, or thinking with words. On this understanding, a speech act/language act does not require anyone to speak out loud, and it does not require that the speech actor have an audience. Since the word 'speech' does suggest speaking out loud, ordinarily with an audience or addressee, the phrase 'language act' is probably more appropriate than 'speech act';

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however, I use these expressions interchangeably. Language acts are also performed by the person who reads or who listens with understanding.

It is language acts which are the primary bearers of such semantic features as meaning and truth. Written and spoken expressions have syntactic features and can themselves be regarded as syntactic objects. Most expressions are conventionally used to perform acts with particular meanings; the meanings commonly assigned to expressions are the meanings of acts they are conventionally used to perform. However, these conventions are not the source of the meanings of meaningful acts, for the language user's intentions determine the meanings of his language acts. While it is normal to intend the meanings conventionally associated with the expressions one is using, a person might by mistake produce the 'wrong' word to perform a linguistic act. A child learning language might use 'dog' (or 'doggie') for small four-legged animals, including cats; the child's use is wrong only because it fails to match our own.

A logical treatment of language acts gives some prominence to *sentential* acts, those language acts performed with complete sentences. Among sentential acts, *propositional* acts are of particular interest—these are the sentential acts that are true or false. Because it is a little cumbersome to say 'propositional act' repeatedly, I also call these acts *statements*. This is a stipulated meaning for 'statement' and is different from its more common use to mean something like 'assertion'. Sentential acts can be performed with one or another *illocutionary force*, and so constitute *illocutionary acts*, such as assertions, requests, and promises. Statements themselves can be performed with various illocutionary forces: a statement can be asserted or denied, it can be supposed true or supposed false. Attention to language acts leads to an expanded conception of logic, for in addition to considering truth conditions, it proves necessary to consider semantic features due to illocutionary force. These features have an important bearing on whether an argument is satisfactory, and this has not previously been noted.

Illocutionary logic makes it possible to accommodate both the ontological and the epistemic dimensions of logic, without reducing either to the other, or emphasizing one at the expense of the other. It does this by adding another layer, or tier, to conventional logical systems.

A standard system of logic contains three elements:

(1) an artificial language;
(2) a semantic account for the artificial language, an account which normally spells out the truth conditions of sentences in the artificial language;
(3) a deductive system which codifies certain logically-important items in the artificial language.

Systems of illocutionary logic differ from standard logical systems in three respects:

(1) Illocutionary force indicating expressions, or, simply, *illocutionary operators*, are added to the artificial language.
(2) The account of truth conditions, the truth-conditional semantics for the artificial language, is supplemented by an account of commitment conditions; these determine what statements a person is committed to accept or reject once they accept and reject some to begin with.

(3) The deductive system is modified to take account of, and accommodate, illocutionary operators.

Artificial logical languages are not used to perform speech acts/language acts. We do not communicate or think with sentences of these languages. Sentences in artificial languages are best construed as representations of natural-language statements. The truth conditions and commitment conditions provided for artificial languages are really for the statements represented by these languages.

An argument conceived as a speech act is an act of reasoning from premiss acts which are assertions, denials, or suppositions to a conclusion which is also an illocutionary act and which is thought to follow from, or be supported by, the premiss acts. Arguments are simple or complex. An argument conceived as a speech act is not appropriately regarded as valid or invalid—it is instead *deductively correct* or not. A simple argument is deductively correct if any person who performs the premiss acts is committed to perform the conclusion act. A complex act is deductively correct if the component arguments are deductively correct, and any person who performs the initial premiss acts is committed to perform the final conclusion act. Proofs or deductions in a natural deduction system can serve as perspicuous representations of deductively correct speech-act arguments.

## 2. Base system

We shall begin with a simple system of propositional logic. The language $L$ contains denumerably many atomic sentences, together with compound sentences formed with '$\sim$', '$v$', and '$\&$'. The horseshoe '$\supset$' of material implication is a defined symbol. The atomic sentences, and the compound sentences formed with connectives are *plain* sentences of $L$; there are no other plain sentences. Plain sentences represent statements.

The language $L$ contains four *illocutionary operators*:

(1) $\vdash$: the sign of assertion;
(2) $\dashv$: the sign of denial;
(3) $\llcorner$: the sign of supposing true (of positive supposition);
(4) $\neg$: the sign of supposing false (of negative supposition).

If an illocutionary operator is prefixed to a plain sentence of $L$, the result is a *completed* sentence. There are no other completed sentences. (So, for example, $\vdash\vdash A$ and $\llcorner\vdash A$ are not well-formed expressions. Neither is '$\vdash[\llcorner A \ \& \ \vdash B]$'.) Completed sentences represent illocutionary acts.

A statement can be accepted or rejected. A person performs an act when they come to accept a statement. Once they have come to accept it, they continue to accept the statement until they change their mind or forget that they have come to accept the statement. Continuing to accept a statement is not an act. A person who accepts a statement can perform an act of reaffirming the statement, or, as I prefer to say, an act *reflecting their continued acceptance of* the statement. We will understand an *assertion* to be an act of producing and coming to accept a statement, or of producing and reflecting one's acceptance of the statement (an assertion of this sort does not need an audience). A person can be committed to come to accept a statement or to continue to accept a statement, but for the sake of simplicity, I will describe such a person as being committed to accept the statement.

In coming to accept a statement, or continuing to accept a statement, we are not characterizing that statement. We are accepting that things are as the statement says. I shall say that we are *accepting the statement as the way things are*. Accepting a statement is different from, and simpler than, characterizing the statement as true. To characterize a statement as true involves greater sophistication and self-consciousness than it does to simply accept the statement. All assertions in our sense are sincere. Ordinarily, an assertion is an act aimed at an audience, in which the speaker presents themselves as, and takes responsibility for, accepting the statement. As customarily understood, an assertion can be insincere. However, a sincere assertion as customarily understood is also an assertion in our sense. Our use of the assertion sign is close to Frege's, for we are using the assertion sign to mark a judgement on the part of the language user.

We will also understand denial in a somewhat idiosyncratic way. A denial is an act of coming to reject a statement, or an act reflecting one's continuing to reject the statement. The sign of denial, '$\dashv$', represents the force of rejecting. A statement is denied for being at odds with the way things are, but a simple denial does not characterize the statement. Just as characterizing a statement as true is more sophisticated, and more demanding, than simply accepting the statement, so characterizing a statement as false, or as not true, is more sophisticated, and more demanding, than simply rejecting the statement.

In ordinary English, a denial might be performed with a sentence like this:

I deny that Milwaukee is in Illinois.

although this would be a little out of the ordinary. It is more natural (more idiomatic) to use a sentence like this:

Milwaukee is not in Illinois.

to deny that Milwaukee is in Illinois. Although this sentence might be used (on different occasions) to perform acts with different semantic structures, one very common way to use the sentence has the word 'not' indicating, and carrying, the force of rejection. The word 'not' is used to block, or bar, the predication of 'is in Illinois' for Milwaukee. By blocking the predication, the language user blocks the statement and its acceptance.

If we simply deny a statement, we have not characterized that statement, but if we negate a statement, we *have* characterized it. To negate a statement is to characterize the statement as being at odds with the way things are. This is what it is for the statement to be false. (In characterizing the statement, we are not characterizing the sentence used to make the statement.) Negating a statement, and accepting the negative statement, are different from simply denying the negated statement, but the two acts (accepting a negative statement and denying the negated statement) often 'come to the same thing'.

Denying a statement has a different 'feel' from asserting a negated statement. One can, perhaps, get an appreciation of this difference by comparing the following sentences and the language acts one might perform with them:

(1) Milwaukee is not in Illinois.
(2) Either Milwaukee is not in Illinois or Chicago is not in Illinois.

Sentences like (1) are 'suited' for making denials; this is the natural way to use them and to understand what someone has done with them. While a disjunction can be used to make an assertion or denial, the disjuncts will not themselves be asserted or denied. It is easy to read and understand sentence (1), but (2) requires more 'work'. (2) might strike a person as awkward, or difficult—to use and understand it, the two disjuncts must be understood as negative statements.

A statement which is accepted is accepted *as true* or as *characterizing things as they are*. But an assertion does not predicate truth of the statement, or explicitly characterize the statement as being true. The statement is simply presented, or taken, as being the way things are. In supposing-true a statement, one is not supposing a statement which predicates truth of an 'inner' statement. One is simply supposing, for the time being, that the statement is how things are. Negatively supposing a statement (supposing-false the statement) is temporarily blocking the statement's acceptance.

We commonly say *suppose this* or *suppose that*, and then reason to the consequences of what we have supposed. When we reach a conclusion based on an initial supposition, we do not usually prefix the conclusion with the word 'suppose'. However, in such a case, the conclusion has the status of a supposition. (We are not entitled to assert it.) I will say that both the initial act and the conclusion are suppositions, and will use the sign of supposition in representing both acts.

We will provide a semantic account and a deductive system for *L*, and establish that the system is adequate with respect to the semantic account. Once we obtain these results, we will proceed to discuss the Liar Paradox and show how illocutionary logic provides a new perspective on this paradox.

## 3. Semantic account

The semantic account for the language *L* is a two-tier account. The first tier applies to statements apart from illocutionary force. This semantic account gives truth conditions of plain sentences and of the statements that these represent. The first tier of the semantic account presents the *ontology* that the statements encode or represent. The account of truth conditions for plain sentences of *L* is entirely standard. An *interpreting function for L* is a function *f* which assigns truth and falsity to the atomic plain sentences, and determines a *truth-value valuation* of the plain sentences in which compound sentences have truth-table values.

The second tier of the semantic account applies to completed sentences and the illocutionary acts they represent. In the case of *L*, it applies to assertions, denials, and suppositions. The second tier of the semantics deals with *rational commitment*. It is somewhat unfortunate from my point of view that the word 'commitment' is used by philosophers and logicians in many different ways. For example, in *Walton* (*1995*, *1999*), we find a concept of commitment that has some features in common with my concept but which is on the whole quite different from my concept. In *Vanderveken* (*2004*), three different concepts of commitment are discussed. I am convinced that my concept of commitment is the one that actually figures in our deductive inferential practice. Since everyone seems to have their own concept, I will explain the concept I have in mind without trying to appeal to some standard meaning of 'commitment'.

The commitment involved is *rational* commitment, as opposed to moral or ethical commitment. This is a commitment to do or not do something. It can also be a commitment to continue in a certain state, like the commitment to continue to accept or reject a given statement. Deciding to do $X$ rationally commits a person to doing $X$. If, before going to work, I decide to buy gasoline on the drive to work, I am committed to doing this. But if I forget, or change my mind, and drive straight to work without buying gasoline, I have not done anything that is morally wrong. I may kick myself for being stupid, or forgetful, but this is not a moral failing. Decisions generate commitments, but performing one act can also commit a person to perform others. For example, accepting the statement that today is Wednesday will commit a person to accept (or continue to accept) the statement that tomorrow is Thursday. The rational commitment that I am considering is quite similar to what Vanderveken calls *weak commitment* in *Vanderveken* (*1990*, *2004*). He contrasts this with a *strong commitment* which I find to be of little interest.

Some commitments are 'come what may' commitments, like my commitment to buy gasoline on the way to work. Others are conditional, and only come up in certain situations, like the commitment to close the upstairs windows if it rains while I am at home. When I accept the statement that today is Wednesday, the commitment to accept the statement that tomorrow is Thursday is conditional. I am committed to do it only if the matter comes up, and I choose to give it some thought. (And I can 'lose' the commitment if I change my mind about my initial assertion.)

The rational commitment that is of concern here is a commitment to act or to refrain from acting, or to continue in a state of accepting or rejecting a statement. Some writers speak of being committed to the truth of some statement, but that is not the present sort of commitment. However, a person might in my sense be committed to acknowledge or admit the truth of a certain statement.

To actually accept or reject a statement, a person must consider the statement and 'take a stand' about the statement. No one can actually accept all the statements they are committed to accept, or reject all those they are committed to reject. A deductively correct argument which begins with assertions and denials can lead a person to expand the class of statements they explicitly accept or those they explicitly reject. Such an argument begins with explicit beliefs and disbeliefs, and traces commitments to produce more explicit beliefs or disbeliefs.

Commitment provides the 'motive power' which propels someone from the premisses to the conclusion of an argument. The premisses and the conclusion are illocutionary acts. The person who makes or who follows an argument needs to recognize (or think they do) a rational requirement to perform the conclusion act. If, for example, the conclusion is an assertion $\vdash A$, then if the argument commits a person to accept or continue to accept $A$, we shall understand that the arguer is committed to perform the act $\vdash A$ (to perform the act represented by the completed sentence). An argument may be such that the truth of its premisses 'requires' the truth of the conclusion. But unless an arguer recognizes the connection between premisses and conclusion, accepting or supposing the premisses will not lead them to accept or suppose the conclusion. It is their recognition that their premiss acts commit them which moves them to perform the conclusion act.

A commitment to perform or not perform an act is always *someone's* commitment. We develop the commitment semantics for an idealized person called the *designated subject*. This subject has beliefs and disbeliefs which are *coherent* in the sense that the beliefs might all be true and the disbeliefs all false. The second tier of the semantics concerns *epistemology* rather than ontology, but the epistemology must accommodate the ontology. The commitments generated by performing certain illocutionary acts depend on the language user understanding the truth conditions of the statements they assert, deny, or suppose. We consider the designated subject at some particular moment. There are certain statements which they have considered and accepted, which they remember and continue to accept. There are similar statements that they have considered and rejected. These *explicit* beliefs and disbeliefs commit them, at that moment, to accept further statements and to reject further statements. We use '+' for the value of assertions and denials that they are committed, at that moment, to perform.

A *commitment valuation* is a function which assigns + to some of the assertions and denials in $L$. A commitment valuation $\mathscr{E}$ is *based on* an interpreting function $f$ if, and only if (from now on: iff), (i) If $\mathscr{E}(\vdash A) = +$, then $f(A) = \mathrm{T}$, and (ii) If $\mathscr{E}(\dashv A) = +$, then $f(A) = \mathrm{F}$. A commitment valuation is *coherent* iff it is based on an interpreting function.

Let $\mathscr{E}_0$ be a coherent commitment valuation. This can be understood to register the designated subject's explicit beliefs and disbeliefs at a given time. The *commitment valuation determined by* $\mathscr{E}_0$ is the function $\mathscr{E}$ such that (i) $\mathscr{E}(\vdash A) = +$ iff $A$ is true for every interpreting function on which $\mathscr{E}_0$ is based, and (ii) $\mathscr{E}(\dashv A) = +$ iff $A$ is false for every interpreting function on which $\mathscr{E}_0$ is based. The valuation $\mathscr{E}$ indicates which assertions and denials the designated subject is committed to perform on the basis of their explicit beliefs and disbeliefs.

A commitment valuation is *acceptable* iff it is determined by a coherent commitment valuation. The following matrices show how acceptable commitment valuations 'work': In the matrices, the letter '*b*' stands for *blank*—for those positions in which no value is assigned (See table 1).

In the first row of the table, we consider the case where the designated subject is committed to accept/assert both $A$ and $B$. They are not committed to reject either statement. In that case, they are committed to deny the negation of each statement, and to accept both the conjunction and the disjunction of the two statements. They are not committed to accept the negation of either statement or to reject the conjunction or the disjunction of the two statements. The other rows are understood in a similar way.

In some cases, the values (or non-values) of assertions and denials of simple sentences are not sufficient to determine the values of assertions and denials of compound sentences. For example, if $\dashv A$ and $\dashv B$ have no value, and $A$, $B$ are irrelevant to one another, then '$\dashv[A \mathbin{\&} B]$' should have no value. But if $\dashv A$, $\dashv\sim A$ have no value, the completed sentence '$\dashv[A \mathbin{\&} \sim A]$' will have value +.

Table 1. Acceptable commitment valuations.

| $\vdash A$ | $\vdash B$ | $\dashv A$ | $\dashv B$ | $\vdash\sim A$ | $\dashv\sim A$ | $\vdash[A \mathbin{\&} B]$ | $\dashv[A \mathbin{\&} B]$ | $\vdash[A \vee B]$ | $\dashv[A \vee B]$ |
|---|---|---|---|---|---|---|---|---|---|
| + | + | *b* | *b* | *b* | + | + | *b* | + | *b* |
| + | *b* | *b* | *b* | *b* | + | *b* | *b* | + | *b* |
| + | *b* | *b* | + | *b* | + | *b* | + | + | *b* |
| *b* | + | *b* | *b* | *b* | *b* | *b* | *b* | + | *b* |
| *b* | *b* | *b* | *b* | *b* | *b* | *b* | +,*b* | +,*b* | *b* |
| *b* | *b* | *b* | + | *b* | *b* | *b* | + | *b* | *b* |
| *b* | + | + | *b* | + | *b* | *b* | + | + | *b* |
| *b* | *b* | + | *b* | + | *b* | *b* | + | *b* | *b* |
| *b* | *b* | + | + | + | *b* | *b* | + | *b* | + |

## 4. Some semantic concepts

The truth conditions of a statement determine what the world must be like for the statement to be true. Many statements can be made true in different ways. For example, the statement:

Some man (or other) is a geologist.

can be made true by different men—for each man, his being a geologist would make the statement true. The truth conditions of a statement seem best regarded as an ontological or *ontic* feature of the statement, if the ontic is being contrasted with the epistemic. But commitment conditions are epistemic. It is individual people who are committed or not by the statements they accept and reject. The person who makes a meaningful statement must recognize the 'commitment consequences' of their statement if they understand what they are saying. At least, they must recognize the *immediate* commitment consequences, for no one can survey all of the longer-range consequences.

The distinction between truth conditions and commitment conditions gives us occasion to recognize different classes of semantic concepts. Consider entailment and implication. I am using 'entail', 'entailment', etc. for a highly general relation based on the total meanings of the statements involved. In contrast, I will use 'imply', 'implication', etc. for the logical special case of this general relation. The logical special case is identified with respect to the logical forms of artificial-language sentences. The statement:

(1) Every duck is a bird.

both entails and implies:

(2) No duck is a non-bird.

But:

(3) Sara's jacket is scarlet.

entails:

(4) Sara's jacket is red.

without implying (4), for the entailment from (3) to (4) is not based on features uncovered by logical analysis.

We can characterize *truth-conditional entailment* as follows: statements $A_1, \ldots, A_n$ (*truth-conditionally*) *entail* statement $B$ iff there is no way to satisfy the truth conditions of $A_1, \ldots, A_n$ without satisfying those of $B$. This characterization resists being turned into a formal definition. But *truth-conditional implication* can be defined formally: Sentences (of $L$) $A_1, \ldots, A_n$ (*truth-conditionally*) *imply* $B$ iff there is no interpreting function $f$ of $L$ such that $f(A_1) = \cdots = f(A_n) = \mathrm{T}$, while $f(B) = \mathrm{F}$. (If there is implication linking sentences of $L$, then there is implication linking the statements which these sentences represent.)

Let $X$ be a set of plain sentences of $L$ and let $A$ be a plain sentence of $L$. Then $X$ (*truth-conditionally*) *implies* $A$ iff there is no interpreting function of $L$ which assigns T to every sentence in $X$, but fails to assign T to $A$.

Let $A_1, \ldots, A_n$, $B$ be plain sentences of $L$. Then '$A_1, \ldots, A_n/B$' is a *plain argument sequence of* $L$. The sentences $A_1, \ldots, A_n$ are the *premisses*, and $B$ is the *conclusion*. (We also consider argument sequences whose components are statements. A plain argument sequence of $L$ will represent a plain argument sequence whose components are natural-language statements.) A plain argument sequence of $L$ is *truth-conditionally* (*logically*) *valid* iff its premisses truth-conditionally imply its conclusion.

*Illocutionary entailment* links illocutionary acts. If $A_1, \ldots, A_n$, $B$ are (each) assertions, denials, or suppositions, then $A_1, \ldots, A_n$ *deductively require* (*illocutionarily entail*) $B$ iff anyone who performs the acts $A_1, \ldots, A_n$ is committed to perform $B$.

In connection with illocutionary entailment (and implication), we recognize both cases where performing an illocutionary act *generates* commitments to perform further acts, and cases where performing an act *reveals* commitments to perform further acts. For example, a person who comes to accept statement $A$ (they perform act $\vdash A$) is committed by this act to accept the statement '$[A \ v \ B]$'. But a person who uses a singular term to (attempt to) refer to an individual reveals by this act that they are committed to accept the statement that the referent exists. The referring act does not generate the commitment. They must already believe there is a referent before they refer to it. Whether performing an act $\vdash A$ generates a commitment to perform $\vdash B$, or reveals a commitment to perform $\vdash B$, we will say that $\vdash A$ is linked to $\vdash B$ by illocutionary entailment.

*Illocutionary implication* links completed sentences of $L$ and the illocutionary acts that these represent. In order to define illocutionary implication, some preliminary definitions are required.

Let $\mathscr{E}_0$ be a coherent commitment valuation of $L$, $\mathscr{E}$ be the commitment valuation determined by $\mathscr{E}_0$, and $A$ be a completed sentence of $L$ that is either an assertion or denial. Then $\mathscr{E}_0$ *satisfies* $A$ iff $\mathscr{E}(A) = +$.

Suppositions are not assigned values by commitment valuations. But supposing certain statements will commit a person to supposing others. In supposing a statement either true or false, we consider truth values to determine what further statements we are committed to suppose.

Let $f$ be an interpreting function of $L$, and let $A$, $B$ be plain sentences of $L$. Then (i) $f$ *satisfies* $\llcorner A$ iff $f(A) = \mathrm{T}$, and (ii) $f$ *satisfies* $\neg B$ iff $f(B) = \mathrm{F}$.

Let $f$ be an interpreting function of $L$ and $\mathscr{E}$ be a commitment valuation of $L$ based on $f$. Then, $\langle f, \mathscr{E}\rangle$ is a *coherent pair for* $L$.

Let $\langle f, \mathscr{E}\rangle$ be a coherent pair (for $L$), and let $A$ be a completed sentence of $L$. Then, $\langle f, \mathscr{E}\rangle$ *satisfies* $A$ iff either (i) $A$ is an assertion or denial and $\mathscr{E}$ satisfies $A$ or (ii) $A$ is a supposition and $f$ satisfies $A$.

Let $A_1, \ldots, A_n$, $B$ be completed sentences of $L$. Then $A_1, \ldots, A_n$ *logically require* (*illocutionarily imply*) $B$ iff (i) $B$ is an assertion or denial and there is no coherent commitment valuation which satisfies the assertions and denials among $A_1, \ldots, A_n$ but does not satisfy $B$, or (ii) $B$ is a supposition and there is no coherent pair for $L$ which satisfies each of $A_1, \ldots, A_n$, but fails to satisfy $B$.

Let $X$ be a set of completed sentences of $L$ and let $A$ be a completed sentence of $L$. Then, $X$ *logically requires* $A$ iff (i) $A$ is an assertion or denial, and there is no coherent commitment valuation which satisfies the assertions and denials in $X$ but does not satisfy $B$, or (ii) $B$ is a supposition and there is no coherent pair for $L$ which satisfies every sentence in $X$ but fails to satisfy $A$.

It is necessary to have two clauses in the definitions of illocutionary implication, because if $B$ is an assertion or denial, its value is independent of the values assigned to suppositions. For example, consider these completed sentences:

$$\sqcup A,\ \neg A,\ \vdash B;\ \vdash [B\ \&\ A].$$

There is no coherent pair which satisfies $\sqcup A$, $\neg A$, $\vdash B$ and fails to satisfy '$\vdash[B\ \&\ A]$', because there is no coherent pair which satisfies $\sqcup A$, $\neg A$, $\vdash B$. However, the first three sentences do not logically require '$\vdash[B\ \&\ A]$', for suppositions make no 'demands' on assertions and denials. Incoherent suppositions logically require that we suppose true and suppose false every plain sentence, but they do not require that we assert or deny anything.

Let $A_1, \ldots, A_n$, $B$ be completed sentences of $L$. Then, '$A_1, \ldots, A_n \rightarrow B$' is an *illocutionary argument* sequence—for convenience, I will simply say that it is an *illocutionary sequence*. We can define a concept of *illocutionary validity* that applies to illocutionary sequences. An illocutionary sequence '$A_1, \ldots, A_n$' is *logically connected* (*illocutionarily logically valid*) iff $A_1, \ldots, A_n$ logically require $B$.

I will use the words 'consistent' and 'coherent' for semantic ideas rather than syntactic or proof-theoretic ideas. Let $X$ be a set of plain sentences of $L$. Then, $X$ is *consistent* iff there is an interpreting function $f$ of $L$ for which every sentence in $X$ has value T. (The sentences have the value T for the *valuation determined by $f$*).

Let $X$ be a set of completed sentences of $L$. This set is *coherent* iff there is a coherent pair $\langle f, \mathscr{E}\rangle$ for $L$ which satisfies every sentence in $X$.

## 5. Deductive system *S*

This is a natural deduction system which employs tree proofs (tree deductions). Each step in one of these proofs/deductions is a completed sentence. An initial step in a tree proof is an assertion $\vdash A$, a denial $\dashv A$, a positive supposition $\sqcup A$, or a negative supposition $\neg A$. An initial assertion or denial is not a hypothesis of the proof. Instead, an initial assertion or denial should express knowledge or justified (dis)belief of the arguer. Not every sentence $\vdash A$ is eligible to be an initial assertion in a proof constructed by a given person. In contrast, any supposition can be an initial supposition. Initial suppositions are hypotheses of the proof.

The elementary rules governing the connectives are the following:

*& Introduction*

$$\frac{\vdash/\sqcup A \quad \vdash/\sqcup B}{\vdash/\sqcup [A\ \&\ B]}$$

*& Elimination*

$$\frac{\vdash/\sqcup [A\ \&\ B]}{\vdash/\sqcup A} \qquad \frac{\vdash/\sqcup [A\ \&\ B]}{\vdash/\sqcup B}$$

*v Introduction*

$$\frac{\vdash/\sqcup A}{\vdash/\sqcup [A\ v\ B]} \qquad \frac{\vdash/\sqcup B}{\vdash/\sqcup [A\ v\ B]}$$

The expression '$\vdash/\sqcup$' is used to indicate that the illustration applies to both '$\vdash$' and '$\sqcup$.' If one premiss is a supposition, then so is the conclusion. Otherwise, the conclusion is an assertion.

The horseshoe of material implication is a defined symbol; it is characterized by *Modus Ponens* ($\supset$ *Elimination*):

$$\frac{\vdash/\sqcup A \qquad \vdash/\sqcup[A \supset B]}{\vdash/\sqcup B}$$ The conditions are the same as for the rules illustrated above.

This is a derived rule of the system $S$.

The following arguments are correct:

$$\frac{\sqcup A \quad \sqcup B}{\sqcup[A \ \& \ B]} \qquad \frac{\vdash A \quad \sqcup B}{\sqcup[A \ \& \ B]} \qquad \frac{\vdash A \quad \vdash B}{\vdash[A \ \& \ B]}.$$

But even though they are truth preserving, these arguments are not correct:

$$\frac{\sqcup A \quad \sqcup B}{\vdash[A \ \& \ B]} \qquad \frac{\sqcup A \quad \vdash B}{\vdash[A \ \& \ B]}.$$

Supposing the premisses commits us to supposing the conclusion, but the suppositions do not authorize us to assert the conclusion.

A deduction in $S$ from initial (uncancelled) sentences $A_1, \ldots, A_n$ to conclusion $B$ establishes that $A_1, \ldots, A_n$ logically require (illocutionarily imply) $B$. It also establishes that the illocutionary sequence '$A_1, \ldots, A_n \rightarrow B$' is logically connected. We can regard the *theorems* of $S$ as illocutionary sequences established by deductions in $S$.

The following proof:

$$\cfrac{\cfrac{\vdash A \quad \vdash B}{\vdash[A \ \& B]}\,\&I \qquad \vdash[[A \ \& B] \supset C]}{\vdash C}\,MP$$

shows that $\vdash A$, $\vdash B$, $\vdash[[A \ \& \ B] \supset C]$ logically require $\vdash C$. It also establishes that the illocutionary sequence '$\vdash A, \vdash B, \vdash[[A \ \& \ B] \supset C] \rightarrow \vdash C$' is logically connected, and is a theorem of $S$.

The rule *Weakening* is another elementary rule of $S$; it has two forms:

$$\frac{\vdash A}{\sqcup A} \qquad \frac{\dashv A}{\neg A}.$$

Accepting or rejecting $A$ establishes $A$'s status once and for all (it is intended to do this, even if one later finds it necessary to change one's mind). Supposing $A$ true or false is putting $A$ 'in play' or 'out of play' for a time, temporarily. It is for the time being accorded a status like that of the accepted or rejected statements. That is the rationale for *Weakening*.

In a standard system of logic, we cannot mark the difference between assertions and suppositions. In a standard natural-deduction system, each step in a proof from hypotheses amounts to a supposition. A proof from initial sentences $A_1, \ldots, A_n$ to conclusion $B$ will, in effect, establish an illocutionary sequence '$\llcorner A_1, \ldots, \llcorner A_n \rightarrow \llcorner B$' to be logically connected. To use a system of standard logic to explore proofs (deductions) which have both hypotheses and initial assertions, we must give some extralogical statements the status of axioms (these can function as initial assertions).

The non-elementary rules for connectives are the following:

*v Elimination*

$$\frac{\vdash/\llcorner[A\ v\ B] \quad \overset{\{\llcorner A\}}{\llcorner C} \quad \overset{\{\llcorner B\}}{\llcorner C}}{\vdash/\llcorner C}$$

*$\supset$ Introduction*

$$\frac{\overset{\{\llcorner A\}}{\llcorner B}}{\vdash/\llcorner[A \supset B]}$$

Non-elementary rules cancel, or discharge, occurrences of the (positive) hypotheses enclosed in braces. If the only uncancelled hypotheses leading to the conclusions on the line are those in braces, then the conclusion is an assertion. Otherwise, it is a (positive) supposition.

The following deduction:

$$\dfrac{\vdash[A\ v\ B] \qquad \dfrac{\overset{x}{\llcorner A} \quad \llcorner[A \supset B]}{\llcorner B}\,MP \qquad \overset{x}{\llcorner B}}{\llcorner B}\ v\ E,\ cancel\ A,\ B$$

establishes that '$\vdash[A\ v\ B], \llcorner[A \supset B] \rightarrow \llcorner B$' is logically connected. An '$x$' is placed above cancelled hypotheses.

There are so far no rules for reasoning with the negation sign. That is because the negative illocutionary acts, and the negative illocutionary force indicators, are regarded as more primitive than, and prior to, the sign of negation. I think that denial is prior to negation in the literal sense that people performed the illocutionary act of denial before it occurred to them to negate propositional acts, which negated propositional acts are then available to be asserted. Even now, I think it is much more common to use a sentence like 'Milwaukee is not in Illinois' to block the application of 'is in Illinois' to Milwaukee than it is to negate the statement that Milwaukee is in Illinois, and assert that statement.

The historical claim that languages developed in such a way that people denied statements before the resources were available to assert negated statements is one for which it might be possible to obtain evidence, but I will not look for such evidence. Nor will I investigate children's acquisition of language to see if we can determine that children deny statements before they learn to negate statements. What I will do here is show that if denial and negative supposition and principles for reasoning with denial and supposition are available, then a sign of negation can be introduced and explained in a somewhat definitional way in terms of negative illocutionary acts.

Rules that are commonly associated with the negation sign are here assigned to negative illocutionary operators:

*Negative Force Introduction*

$$\frac{\begin{array}{ll}\{\llcorner A\} & \\ \llcorner B & \dashv/\neg B\end{array}}{\dashv/\neg A} \qquad \frac{\begin{array}{ll} & \{\llcorner A\} \\ \vdash/\llcorner B & \neg B\end{array}}{\dashv/\neg A} \qquad \frac{\begin{array}{ll}\{\llcorner A\} & \{\llcorner A\} \\ \llcorner B & \neg B\end{array}}{\dashv/\neg A}$$

If the only uncancelled hypothesis leading to the sentences above the line is the one in braces, then the conclusion is '$\dashv A$'; otherwise, it is '$\neg A$'.

*$\neg$Elimination*

$$\frac{\begin{array}{ll}\{\neg A\} & \\ \llcorner B & \dashv/\neg B\end{array}}{\vdash/\llcorner A} \qquad \frac{\begin{array}{ll} & \{\neg A\} \\ \vdash B & \neg B\end{array}}{\vdash/\llcorner A} \qquad \frac{\begin{array}{ll}\{\neg A\} & \{\neg A\} \\ \llcorner B & \neg B\end{array}}{\vdash/\llcorner A}$$

If the only uncancelled hypothesis leading to the sentences above the line is the one in braces, then the conclusion is '$\vdash A$'; otherwise, it is '$\llcorner A$'.

The principle/rule $\neg$*Elimination* is understood in such a way that the following is an instance of the principle:

$$\frac{\begin{array}{lll} x & & \\ \neg A & \vdash/\llcorner B & \dashv/\neg B\end{array}}{\vdash/\llcorner A}$$

A similar remark applies to the principle *Negative Force Introduction*.

Given these principles for negative illocutionary operators, we can now provide the inferential features of the negation sign.

*$\sim$Introduction* *$\sim$Elimination*

$$\frac{\dashv A}{\vdash\sim A} \qquad \frac{\neg A}{\llcorner \sim A} \qquad\qquad \frac{\vdash\sim A}{\dashv A} \qquad \frac{\llcorner \sim A}{\neg A}$$

The sign of negation allows us to 'disentangle' falsity conditions from the force of negative illocutionary acts. A positive assertion '$\vdash\sim A$' comes to much the same thing as the denial '$\dashv A$.' The negation sign also makes possible completed sentences that cannot be 'translated' into sentences with negative illocutionary prefixes. An example is: '$\vdash[A\ v\ \sim B]$'.

We prove '$\vdash\sim\sim A \rightarrow \vdash A$' and '$\llcorner\sim\sim A \rightarrow \llcorner A$' to be logically connected as follows:

$$\cfrac{\cfrac{\overset{x}{\neg A}}{\llcorner \sim A}\,\sim I \qquad \cfrac{\vdash/\llcorner\sim\sim A}{\dashv/\neg\sim A}\,\sim E}{\vdash/\llcorner A}\,\neg E,\ \textit{cancel}\ \neg A$$

Given the principles *¬Elimination, ~Elimination,* and *~Introduction*, the principle *Negative Force Introduction* is a derived rule. But this principle will be retained as a primitive rule, for we are considering the language without the sign of negation to be the predecessor and source of the language which contains negation. When there is no negation sign, the principle *Negative Force Introduction* is (still) a correct principle, and it is not at that point a derived rule.

It is a straightforward matter to establish that the system $S$ is sound and complete in appropriate illocutionary senses. I will sketch these arguments below.

**Lemma 5.1.** Let $\Gamma$ be a proof in $S$ from initial (uncancelled) sentences $A_1, \ldots, A_n$ *to conclusion* $B$. Let $\langle f, \mathscr{E}\rangle$ be a coherent pair for $L$ which satisfies each of $A_1, \ldots, A_n$. Then this pair satisfies $B$.

This is proved by induction on the *rank* (the number of steps) of $\Gamma$.

**Theorem 5.2.** Let $\Gamma$ be a proof in $S$ from initial sentences $A_1, \ldots, A_n$ to conclusion $B$. Then $A_1, \ldots, A_n$ logically require $B$.

Let $X$ be a set of completed sentences of $L$. $X$ is *deductively coherent with respect to* $S$ iff there is no plain sentence $A$ of $L$ such that both $\llcorner A$ and $\neg A$ can be deduced in $S$ from initial sentences in $X$. The set $X$ is *maximally deductively coherent with respect to* $S$ iff $X$ is deductively coherent with respect to $S$ and for every supposition $A$ (either $\llcorner B$ or $\neg B$) of $L$, either $A \in X$ or $X \cup \{A\}$ is not deductively coherent with respect to $S$.

**Lemma 5.3.** Let $X$ be a set of completed sentences of $L$ that is deductively coherent with respect to $S$. Then, $X$ can be extended to a set $Y$ that is maximally deductively coherent with respect to $S$.

**Lemma 5.4.** Let $Y$ be a set of completed sentences of $L$ that is maximally deductively coherent with respect to $S$. Let $A$, $B$ be plain sentences of $L$. Then, (i) $\llcorner A \in Y$ iff $\neg A \notin Y$ iff $\llcorner \sim A \notin Y$; (ii) $\llcorner[A\ \&\ B]$ iff $\llcorner A \in Y, \llcorner B \in Y$; (iii) $\llcorner[A\ v\ B] \in Y$ iff either $\llcorner A \in Y$ or $\llcorner B \in Y$.

**Lemma 5.5.** Let $Y$ be a set of completed sentences of $L$ that is maximally deductively coherent with respect to $S$. Let $f$ be a function defined on the atomic plain sentences of $L$ such that $f(A) = \mathrm{T}$ iff $\llcorner A \in Y$, and $f(A) = \mathrm{F}$ otherwise. Let $\mathscr{E}_0$ be a function defined on the assertions and denials of $L$ such that $\mathscr{E}_0(A) = +$ iff $A \in Y$. Then $f$ is an interpreting function of $L$. In the valuation determined by $f$, a plain sentence $A$ has value T iff $\llcorner A \in Y$. And $\mathscr{E}_0$ is a commitment valuation based on $f$, as is the commitment valuation determined by $\mathscr{E}_0$.

**Theorem 5.6.** Let $X$ be a set of completed sentences of $L$, and let $A$ be a completed sentence of $L$ such that $X$ logically requires $A$. Then, there is a proof of $A$ in $S$ from initial sentences in $X$.

If $A$ is a supposition $\llcorner B$ or a supposition $\neg B$, we argue in a completely standard way to establish that $A$ can be derived from premisses in $X$.

Suppose $A$ is an assertion $\vdash B$. Let $|X|$ be the subset of $X$ whose members are the assertions and denials in $X$. Suppose that $A$ is not deducible from premisses in $|X|$. Then $|X| \cup \{\neg B\}$ is deductively coherent with respect to $S$, for otherwise $\vdash B$ is deducible from premisses in $X$. But then $|X| \cup \{\neg B\}$ can be extended to a maximally deductively coherent set with respect to $S$, which set determines both an interpreting function and a commitment valuation based on that function. The commitment valuation is based on the interpreting function, and assigns $+$ to the assertions and denials in $X$. But this commitment valuation does not satisfy $A$. This is impossible. Hence, $A$ is deducible from premisses in $X$.

If $A$ is a denial $\dashv B$, the argument is similar.

## 6. Negation and negative force

The conjecture that denial is prior to negation in the sense that people performed speech acts of denial before it occurred to them to negate statements is not one that I am in a position to prove, or even one for which I can provide empirical evidence. But the system developed in this paper shows that denial could be prior, for we can use the inferential principles which characterize denial and negative supposition to introduce and completely characterize negation. I think it plausible that denial is prior to negation, because negating seems more complicated, and more sophisticated, than simply denying. Denial is probably also temporally prior to *supposing* statements either true or false. Supposing is also a relatively sophisticated practice.

My conjecture is that language users first asserted some things and denied others, without considering force-neutral statements and wondering if they were true or false. Language users denied statements before they understood that they could dissociate the illocutionary force from the negative character of the statement. This more sophisticated understanding made it possible to negate statements, and to assert negative statements. On this conjecture, the significance of '$\sim$' is derived from the acts performed with '$\dashv$' and '$\neg$'. The act signalled (represented) by '$\dashv$' *blocks* or *bars* the assertion of a statement, or the use of a predicate to acknowledge one or more individuals to satisfy the predicate's criterion (e.g. 'Charles is *not* rich'). (We use '$\neg$' temporarily to block the statement.)

In an approach to language and logic that takes speech acts to be the fundamental linguistic reality, it seems highly appropriate to try to understand denial and negation in terms of what a person is doing when they deny a statement or negate one. We do not simply want an abstract account of truth conditions. Instead, we want to know what is going on. Assertion seems to be fundamental. We use words to identify things and to characterize them on the basis of their satisfying certain criteria. In denying a statement, we do not consider a force-neutral statement and characterize it as false. By blocking or barring the assertion of the statement, we are keeping the assertion from happening. It requires greater sophistication to merely consider a force-neutral statement than it does to assert or deny a statement. It requires more sophistication to consider what it is for a statement to deserve rejection than it does to simply reject the statement.

The idea that an expression commonly associated with a connective or operator in a logical language might be used to express illocutionary force has an application to other expressions than negative expressions. This helps us understand why, in a natural-deduction system, we can completely characterize the inferential features of operators by rules that apply only when the operators are the principal operators in a sentence, and also to understand why a natural deduction formulation in terms of introduction and elimination rules strikes us as intuitively appropriate. The logical operators are introduced and explained in terms of illocutionary force-indicating expressions. Illocutionary force applies to statements as a whole; it makes no sense to have one illocutionary operator within the scope of another. The inferential features of logical operators are explained in terms of the inferential features of the illocutionary operators from which the logical operators are derived.

## 7. On to the Liar

A common approach when dealing with apparent paradoxes is to look for a mistake which has the nature of a misconception. This has certainly been a favourite approach when dealing with the Liar. One tries to locate a misconception which makes people mistakenly think there is a genuine problem. Either it is not legitimate to even formulate the Liar Statement or, if this is legitimate, the Liar Statement is not one for which questions of truth and falsity arise.

However, I think it would be a mistake to dismiss the Liar Paradox as arising from a simple error or misunderstanding of the rules for correct speaking. Nor does the reasoning involved in the Liar Paradox display any obvious, or even subtle, flaws. In discussing the Liar Paradox, we are concerned with attempts to characterize the statement one is making as false, or untrue. Some discussions of this paradox focus on sentences, and others bring in propositions, but I will stick with statements. From the present perspective, it is language acts which are the primary bearers of semantic properties, and it is statements that 'aspire' to truth and falsity. Various sorts of sentences have been used to make Liar statements. We might have a simple Liar statement, like that made with, or represented by, (a):

(a) Statement (a) is false.

or a 'strengthened' Liar:

(b) Statement (b) is not true.

We can have Liar chains:

(c) Statement (d) is true.
(d) Statement (c) is false (or: not true).

There are contingent Liar statements:

(e) Either *A* or statement (e) is false.

where *A* is some contingent, non-Liar statement. If *A* is true, then the second disjunct is simply false but not paradoxical. If *A* is false, then the second disjunct is (indirectly) characterizing itself as false.

To approach the Liar (or Liars) in the right way, we need to reflect on a feature of illocutionary logic, and on illocutionary acts more generally. In systems of illocutionary logic that we have developed, the semantic accounts have two tiers. The first tier is for statements apart from illocutionary force. The semantic accounts employ interpreting functions which determine valuations of the *plain* sentences of the artificial languages; the plain sentences, which represent statements, are either true or false. The second tier of the semantic accounts is for illocutionary acts and employs valuations which assign commitment values to completed sentences, which represent illocutionary acts.

The distinction between force-free statements (which we also call *propositional acts*), which are in the true–false 'line of work', and illocutionary acts, which are constituted by statements together with illocutionary force (statements performed with illocutionary force), is characteristic of the class of illocutionary acts that Searle (*1985*) calls *assertives*, but not of other illocutionary acts. A statement is true or false independently of the illocutionary force with which it is made (performed). Indeed, it is the truth or falsity of the statement which is the 'concern' of the illocutionary acts of asserting, denying, supposing true, and supposing false.

Someone who makes the following request:

Please close the door.

is not concerned with the sentential act and its features. The speaker performs the sentential act in order to get the addressee to close the door. Even in a language (or language game) where a request is made by saying this:

The door is closed, please make it so.

there is no statement contained in the request. The door is represented as being closed, but this representing act is not a statement which either fits or does not fit the world. Neither the speaker nor the hearer is concerned to evaluate the sentential act.

Statements are *intended* to fit or not fit the world. They are so designed that they can be evaluated as fitting or not. The 'contents' of directives are intended to provide guidance. Statements can be accepted or not (among other things); the contents of directives can be implemented or not. To accept a statement is in some sense to 'keep' it. Implementing a directive allows us to 'discard' that directive and its contents.

In distinguishing locutionary from illocutionary acts, Austin was recognizing something like the statement–illocutionary act distinction for all illocutionary acts. Austin's locutionary–illocutionary distinction has been criticized by a number of philosophers and upheld by others. Searle (*1969*), in particular, has maintained that the distinction is a mistake. Locutionary acts are not distinct acts which can 'stand on their own feet', but are at best abstracted versions of illocutionary acts. Even as abstractions, locutionary acts will have an illocutionary potential which makes such acts far from force-free.

Our position is a kind of compromise. The distinction between the statement made and the force with which it is made is important for assertives. Essentially the same statement can be asserted or denied. That statement can be supposed either true or false. Searle is right that when a statement is made with a certain illocutionary force, there is no separate and independent act which constitutes making the statement. But the statement and its features need to be studied independently from illocutionary force (at the first semantic level). The same can probably not be said about all illocutionary acts.

Asserting a statement is different from giving advice or making a request. We can 'pry apart' the statement from its force. It is more difficult to separate the advice from the giving, or the request from the making. Just as the statement is distinct from the force with which it is made, so the speaker's intention to formulate this statement is distinct from their intention to accept/assert or reject/deny this statement. What they intend to be saying is a different matter from whether they also accept this saying—even if, in practice, they say and accept it all at once.

While statements are characteristically uttered (and written and thought) with a certain force, they are not always used with an illocutionary force of their own. Someone who asserts this disjunctive statement:

> Either Laramie is the capital of Wyoming or else Cheyenne is the capital of Wyoming.

has not asserted either disjunct, although each disjunct is an independent statement. And it is not uncommon to formulate a statement about which we are trying to make up our mind.

## 8. A Paradoxical statement

The Liar Paradox arises at the level of statements. It concerns truth and falsity, not rational commitment. A Liar statement 'says' of itself that it is false, or not true. This 'saying' and the difficulties to which it gives rise are tied to the truth conditions of statements. Some speech-act accounts of the Liar, like that of *Johansson* (*2003*), try to explain the paradox by appealing to features of assertions; but the problem is prior to the second semantic level. However, even though the Liar Paradox is a first-level problem, it also leads to trouble for rational commitment.

Let us consider the inferential puzzles posed by a simple formulation of the Liar. We shall begin with the statement we make (or represent) with this sentence:

> (a) Statement (a) is not true.

Although I have chosen to say 'not true' rather than 'false', I use these expressions interchangeably with respect to statements. A statement is false if it is not true, and it is not true if it is false. Not every statement made with the sentence above will be a Liar Statement. For example, we can imagine someone saying this who had a different statement (a) in mind. Reference and denotation depend to a large extent on the speaker's/writer's intention.

If someone asserts an instance of our standard Liar Statement, they are committed to deny it:

(1) $\vdash$Statement (a) is not true.
(2) $\vdash$It is true that statement (a) is not true. Follows from 1.
(3) $\vdash$Statement (a) is true. This is a paraphrase, or restatement, of 2.
(4) $\dashv$Statement (a) is not true. Follows from 3 (as $\dashv{\sim}A$ follows from $\vdash A$).

It is important to understand the principle of paraphrase involved in the movement from 2 to 3. If $\alpha$ names statement $A$, then this move involves paraphrase:

$$\frac{\vdash/\llcorner\alpha \textit{ is true}}{\vdash/\llcorner \textit{It is true that } A}$$

A move in the opposite direction is also a case of paraphrase. These inferences depend on $\alpha$ naming $A$, and on the 'It is true that' operator expressing the same concept as the predicate 'is true'.

Not only does asserting the Liar Statement commit a person to denying it, but we can infer the assertion from the denial:

(5) $\dashv$Statement (a) is not true.
(6) $\vdash$It is not true that statement (a) is not true. 5, Negation introduction
(7) $\vdash$Statement (a) is not true. 6, paraphrase

It is possible to neither assert nor deny the Liar, and this appears to be what we should do. We simply decline to take a stand about the truth or falsity of this statement. However, even to suppose the Liar Statement leads to trouble:

$$\dfrac{\dfrac{\dfrac{\dfrac{\overset{x}{\llcorner\text{Statement (a) is not true.}}}{\llcorner\text{It is true that statement (a) is not true.}}}{\llcorner\text{Statement (a) is true.}}}{\neg\text{Statement (a) is not true.}} \qquad \overset{x}{\llcorner\text{Statement (a) is not true.}}}{\dashv\text{Statement (a) is not true.}}$$

Supposing statement (a) true leads to a contradiction. By the principle *Negative Force Introduction*, this leads to the denial of statement (a). And this, in turn, will lead to the assertion of statement (a). And so on.

Asserting, denying, or supposing the Liar Statement leads to trouble. In our formulation, the reasoning that ends in incoherence depends on principles involving truth and truth claims. Just as denial is prior to negation, the illocutionary act assertion is prior to, and grounds, acts which characterize a statement as true. Assertion is fundamental, and central to our use of language, but explicit truth claims are not similarly important. For example, if we have an '*It is true that*' operator in our language, the operator will be adequately characterized by these principles:

'*It is true that*' *Introduction*

$$\frac{\vdash/\llcorner A}{\vdash/\llcorner \textit{It is true that } A}$$

'*It is true that*' *Elimination*

$$\frac{\vdash/\llcorner \textit{It is true that } A}{\vdash/\llcorner A}$$

Similarly, if we have an 'is true' predicate to combine with names of sentences (statements), and $\alpha$ names sentence $A$, then we can argue both like this:

$$\cfrac{\cfrac{\vdash/\llcorner A}{\vdash/\llcorner \textit{It is true that } A}}{\vdash/\llcorner \alpha \textit{ is true}}$$

and like this:

$$\cfrac{\cfrac{\vdash/\llcorner \alpha \textit{ is true}}{\vdash/\llcorner \textit{It is true that } A}}{\vdash/\llcorner A}$$

These arguments are the inferential counterpart of Tarski bi-conditionals. Although the principles for the '*It is true that*' operator and the principles of paraphrase are elementary, and statements '*It is true that A*' convey no more information than $A$, this in no way supports a deflationary account of truth. It is essential to a statement that it be true or false; one's intention in making a statement is to say what is true or false. Truth is also essentially bound up with assertion: that $A$ represents things as they are is part of the criterion for correctly asserting $A$. Truth is neither trivial nor unimportant, although characterizing $A$ as true does not add much to $A$. (A deflationary account of truth predication does not support a deflationary account of truth.)

In our formulation, the arguments from '$\vdash/\llcorner\alpha$ *is true*' to '$\vdash/\llcorner A$' and back again are essential for showing the characteristic inferential features of the paradoxical statement. These principles are both innocuous and inescapable. However, if 'false' is taken to be a primitive predicate, and a Liar sentence is formulated like this:

(b) Statement (b) is false.

we can rely on the following principles of paraphrase (where '$\alpha$' still names $A$):

$$\frac{\vdash/\llcorner \alpha \textit{ is false}}{\vdash/\llcorner \sim A} \qquad \frac{\vdash/\llcorner \sim A}{\vdash/\llcorner \alpha \textit{ is false}}$$

to argue from '$\vdash/\llcorner$*Statement (b) is false*' to '$\dashv/\neg$*Statement (b) is false*' and back again. Even in a language without negation, we can argue from '$\vdash/\llcorner$*Statement (b) is* *false*' to '⊢/⌞*Statement (b) is true*', and conversely. None of these inferential principles involving truth and falsity are to blame for the troubles to which the Liar Statement gives rise. It is the statement, not the reasoning, which is faulty.

We cannot coherently either assert or deny the Liar. Even to suppose the Liar true produces incoherence in our beliefs and disbeliefs. Previously, we had presumed that any statement could safely be supposed true. However, in *Greenough* (*1999*, *2001*), Patrick Greenough has argued that the Liar Statement is not *supposition apt*. In *Greenough* (*2001*), there is a long and interesting discussion of what it takes for a statement to be supposition apt. For the present, we can understand his position to be that a significant statement is not supposition apt iff supposing it true leads to the sort of difficulty exemplified by the Liar Statements. Perhaps the right thing to say about Liar Statements is that any language user is committed to decline to assert and to decline to deny such statements, and also to decline to suppose them either true or false.

These troubles involving rational commitment arise at the level of illocutionary acts. But the Liar Statement makes trouble before we reach the second level. The truth conditions of the Liar have this statement being true if, and only if, it is not true. They seem to require the statement to be both true and not true, which might lead us to look favourably on paraconsistent logic, or to recognize some sequential process where the Liar can alternate between being true and being false, as the *Belnap and Gupta* (*1993*) approach suggests. Neither response makes clear sense. We need to take a deeper look at what is involved.

## 9. Intentional acts

Speech acts, or language acts, are intentional acts. These acts are done on purpose, and the agent is aware of what they are doing. The agent need not be aware of every feature that the act possesses, but they must have an intention for their act. The intention *for* an act is different from the intention *of* an act. The intention of an act is what the agent intends to accomplish by performing the act. The intention for an act is what the agent intends for the act to be. The agent who performs an intentional act must intend to perform an act of a certain kind, and they must know what kind this is. In order to make a statement, which is an intentional act, the language user must understand what they are saying or thinking. The statement they accept is the statement that they understand.

Statements are typically complex acts. In making a statement, a person will normally perform what I shall call a *predicative act*. This is not simply an act of uttering a predicate and combining it with an act of using a name or denoting phrase. Any act of combining a predicate with a name or denoting phrase will *represent* an object as satisfying the predicate's criterion. A *predicative act* is intended to be an act which 'confronts' the world, so that it 'fits' those objects that satisfy the predicate's criterion, and fails to fit those that do not. In many cases, the person who performs a predicative act will predicate the feature involved of a particular object. They might also predicate the feature of an unspecified object ('Some woman will be president of the US'), or predicate it of every object of a certain kind. When a person performs a predicative act in making a statement, their intention for the predicative act produces the intention that the statement fit the world or not, that it be true or false. The speaker may further intend to say what is true, but to intend this requires the prior intention to perform the kind of act that is 'up' for being true or false.

A person can represent an object as this or that in a picture. And a person can represent an object as something or other by means of language. We do not assert/accept or deny/reject pictures. We cannot do this, for pictures are not suited for such treatment. (We *can* make and accept the statement that the picture is an accurate representation of an object or event.) Neither is a linguistic representation, merely as such, suitable for being accepted or rejected. A request, for example, may represent a state of affairs, or represent the addressee as performing a certain kind of action, but the request does not contain a statement. What is characteristic of statements is that a statement's maker intends to-fit-or-not-fit the world. The intention to speak truly-or-falsely, which is characteristic of statements, is derived from the intention to perform a predicative act which fits those objects that satisfy the relevant criteria and fails to fit those objects that fail to satisfy the criteria.

If we speak of statements being true or not in ordinary situations, we intend to characterize the statements as fitting the world or not. In making this statement:

What Marcia said about the accident is true.

we are characterizing Marcia's statement or statements as saying of things that are that they are. If we accept our statement, we are endorsing Marcia's claims. Our intention for our *statement* is that it should characterize what Marcia has said (that it characterize this either truly or falsely). Our intention for our *assertion* is to endorse Marcia's claims.

If we consider this version of the Liar:

(a) Statement (a) is not true.

it poses a number of problems. We commonly deny statements by using an internal 'not' to block the application of a predicate to an object. We deny the statement by preventing it from occurring. We cannot properly use the sentence above in this way. For if we block the application of 'true' to statement (a), there is no statement (a). In order that the sentence be used to make a statement, the 'not' must be used to negate a statement rather than to block the application of the predicate. We express the statement that statement (a) is true and characterize this statement as not being true. In negating the statement that statement (a) is true, we have made (performed) statement (a).

The very possibility of making a Liar Statement, or considering the Liar Paradox, depends on there being the distinction between a statement and the illocutionary act performed by making the statement with a certain force. It is the statement that gives us problems, and the statement's problems lead to whatever difficulties we find with assertions, denials, and suppositions. There are no paradoxical requests, for example. In making a request, one does not characterize anything, certainly not the language act one is performing. One can, of course, request someone to do what is impossible, or even contradictory. But if I request someone not to comply with the request I am making, I simply have not made a request. The addressee will be puzzled about what I could possibly have in mind, but there is no paradox. The same thing is true if I advise someone not to follow the advice I am at that moment giving. The correct response is 'What advice?'

I have argued that denial is prior to negation. To deny a statement is to reject a statement for being at odds with the way things are. But to deny a statement is not to characterize that statement, it is not to *say* that the statement is at odds with the way things are. We *do* characterize a statement when we negate the statement. If negation initially derives its significance from its inferential connections with denial, then in negating a statement we characterize the statement as being one that deserves to be denied (we characterize the *statement*, not the sentence that is spoken or written). Upon reflection and analysis, the property of deserving to be denied turns out to be the property of having a bad fit with the way things are. Denial is prior to negation. And falsity is more fundamental than negation. That a statement is false is what makes it not true. To negate a statement is to characterize that statement as false.

In negating a statement, or saying that a statement is false in some other way, one intends to say what is true of statements that are not true, and what is false of statements that are true. In making a statement, one intends to say what is true or false. The Liar Statement frustrates our intentions. In trying to make a statement that is true or false, we end up with a 'statement' that is true if, and only if, it is not true. But this is not possible. Nothing has a property if, and only if, it does not have that property. There is no person who is intelligent if, and only if, they are not intelligent. There is no object that is physical if, and only if, it is not physical. And there is no statement that is true if, and only if, it is not true.

If someone performs an intentional act, they must have an intention for their act: they must intend to perform an act of a certain kind. But we do not always realize (achieve) our intentions. Some intentions cannot be realized. The intention for negation is that in negating a true statement, we will obtain a statement that is not true, and in negating a statement that is not true, we will obtain a true statement. In our formulation, the following is a statement:

Statement (a) is true.

(we can understand this as '*attempted* statement (a) is true'), but it is not true. However, when we negate this:

Statement (a) is not true.

the result is not a true statement. It is not a statement at all, for in negating statement (a), we have performed an act which fails to confront the world in such a way that it either fits or does not fit. It is so designed that it would fit just in case it did not fit. The intention for negation is realized on most occasions when we negate a statement. It cannot be realized when the negation is applied in such a way as to yield a paradoxical attempted statement.

We began by speaking of the Liar Statement, or Liar statements, but we have reached the conclusion that there are no Liar statements. There are Liar sentences, and *attempted* Liar statements, but no actual Liar statements. My position here, and even my analysis, seem to be in general agreement with that developed in *Thalos* (*2005*), though there are many differences of detail. There are other cases of language acts that aspire to be acts of a certain kind, but fail to make the grade. For example, in order for a person to use a singular term $\alpha$ to refer to an object $\gamma$, there must be an object $\gamma$. A person who mistakenly thinks Tarzan to have been a real person can attempt to use 'Tarzan' to refer to Tarzan. We can say that the speaker uses 'Tarzan' in a referring way, but they nonetheless fail to refer. They intend to refer to Tarzan, but their intention cannot be realized. However, admittedly, this case is much less puzzling than the case of the Liar.

It has been a popular strategy in dealing with the Liar Paradox to discredit Liar sentences. They are either ill-formed, or commit category mistakes, or fail for some other reason to be significant or to express propositions. The present account may seem to be just another version of this familiar strategy. However, in addition to being the right account, the present account has some distinctive features. I have not dismissed attempts to characterize what one is presently saying as illicit. I have provided a theoretical framework which does much more work than simply dealing with the Liar but which does explain how the Liar comes about and what goes wrong. And, as I will show in what follows, our account accommodates the practice of formulating Liar sentences and attempting to make Liar statements, without fear of inconsistency or incoherence.

## 10. Avoiding incoherence

Our ordinary practice in using language is not 'precarious'. We are not in danger of drifting into inconsistency or incoherence simply by our normal speaking, writing, and thinking. Neither are we forced to climb an endless hierarchy of metalanguages to avoid contradicting ourselves. We can significantly talk about the language we are using, even about the statements we are making.

The paradoxical character of the attempted Liar Statement 'arises' at the level of truth conditions, for the statement would be true iff it is not; however, the paradoxical character has a distinctive inferential 'signature'. Asserting/accepting the statement would commit a person to deny/reject it, and conversely, supposing the statement true will lead a person to reject it.

If we use our formal languages as a model for understanding the relation between denial (and negative supposition) and negation, and for thinking about how negation might have been introduced, we can see that our intentions for negation must fail to be completely realized. The principles:

$$\begin{array}{ccc} \sim\!\textit{Introduction} & \qquad\qquad & \sim\!\textit{Elimination} \\ \dfrac{\dashv/\neg A}{\vdash/\llcorner \sim A} & & \dfrac{\vdash/\llcorner \sim A}{\dashv/\neg A} \end{array}$$

reflect the view that negating a statement characterizes that statement as one that merits denial, as well as the view that one speaks truly in negating a statement meriting denial. However, in introducing negation into a language, we not only provide the resources for characterizing statements that could already be made in the original language, we also provide the expanded language with new statements that can be negated. The expanded language contains statements like:

$$[A \ v \ \sim B], \quad \sim[\sim A \ \& \ B]$$

which have no counterparts in the original language. The expanded language also enables Liar sentences to be formulated.

The intention for negation is that a statement $\sim A$ will be true iff $A$ is false, and false iff $A$ is true. But a Liar sentence defeats this intention. If we use this formulation:

(a) $\sim$Statement (a) is true.

and understand 'statement (a)' to label *attempted* statement (a), then this statement:

Statement (a) is true.

is not true but false. In negating this false statement, we have characterized it as false. (We have represented or presented it as false.) We have also attempted to say what would be true of a false statement, and false of a true one. This attempt has failed, for if we had said the one, we would have said the other. We have not said anything either true or false: we have not made a statement at all.

When a language in which (for which?) denial is possible is enlarged to accommodate statement negation, which enables us to characterize statements as false, the intention for negation cannot be completely realized. Negation will not always yield a true statement when applied to a false statement. This claim that the attempted Liar Statement defeats our intention for negation has some resemblance to the view articulated in *Eklund* (*2001*, *2002*). In those papers, Eklund argues that our semantic competence leads us to (mistakenly) accept the reasoning in the Liar Paradox. While this competence is just what we need to use language appropriately and correctly in normal situations, it leads us into error when paradox intrudes. However, it is not our competence which gets us into trouble. There *is* reasoning (inference principles) which is correct when dealing with statements that we can determine not to be paradoxical, that would not be correct when employed with (attempted) Liar statements (or with 'statements' that might, for all we can tell, be paradoxical). But we are able to determine which inference principles fit which situations. Our knowledge, including our knowledge of inference principles, does not commit us to accept either the Liar or the statement which the Liar negates. The Liar Paradox 'derives' from our ability to introduce certain expressions, or operations, for which we have intentions that cannot be entirely realized. Such expressions may be quite useful, if our intentions for them can usually, or even often, be realized. We simply need to understand the consequences of failure to realize these intentions.

This understanding may seem to mark agreement with the position of Patrick Greenough that was described above. If an attempted Liar Statement is not a genuine statement, then it cannot appropriately be accepted/asserted or rejected/denied. Even supposing it to be true, or false, is inappropriate. This is Greenough's position, but I think he goes too far.

If a person tries but fails to make a true or false statement, then that person has made an error when they accept or try to accept this statement. They may think that they have made and accepted a genuine statement. They accept that things are as their attempted statement says they are. But things cannot be that way; there is no way for them to be. Their attempted statement is a failed attempt, and their attempt to accept a genuine statement is also a failed attempt. However, things are different for rejection/denial.

We can deny a statement by forming (performing) the statement, and rejecting it—which is to block its acceptance. We more commonly deny a statement by blocking the formation of the statement, rejecting the attempt to make the statement. There is nothing amiss if we also reject an unsuccessful attempt to make a statement. We can both reject a false statement for being false and reject an attempted statement for being a failed attempt. If we now use this sentence:

(a) $\sim$Statement (a) is true.

for our standard Liar sentence, to attempt to perform this act:

$\vdash\sim$Statement (a) is true.

is to try but fail to assert/accept a statement. But this act:

$\dashv\sim$Statement (a) is true.

is entirely in order, as is this one:

$\dashv$Statement (a) is true.

The principle $\sim$*Introduction*, which takes us from:

$$\begin{array}{rl} & \dashv/A \\ \text{to :} & \vdash/\llcorner\sim A \end{array}$$

is not always correct. We can only allow the move if $\sim A$ is a genuine statement. It is correct to reject the unnegated Liar: $\dashv$Statement (a) is true. But we cannot reason by $\sim$*Introduction* in this case, for the following cannot be a correct or successful act:

$\vdash\sim$Statement (a) is true.

We cannot *accept* a failed attempt to make a statement as the way things are. We *can* reject the failed attempt. But what about suppositions: is a failed attempt to make a statement *supposition apt*? A speech act which is a true or false statement can legitimately be supposed true or supposed false. However, there seems to be nothing amiss if we suppose a speech act which is a failed attempt to make a statement to actually be a successful attempt to make a true statement, or a false statement. Consider the following argument from a supposition, using '$\sim$' rather than 'not':

$$\dfrac{\dfrac{\dfrac{\dfrac{\overset{x}{\llcorner\text{Statement (a) is true.}}}{\llcorner\text{It is true that }\sim\text{Statement (a) is true.}}\;\text{Paraphrase}}{\llcorner\sim\text{Statement (a) is true.}}\;\text{‘It is true that’ Elimination}}{\neg\text{Statement (a) is true}}\;\sim\text{Elimination}\qquad\overset{x}{\llcorner\text{Statement (a) is true.}}}{\dashv\text{Statement (a) is true.}}\;\begin{array}{l}\text{Negative}\\ \text{Force Introduction}\end{array}$$

The conclusion is correct. We reject 'Statement (a) is true' for being false. What is not correct, or would not be correct, is the move from this denial to the following: ⊢∼Statement (a) is true. The application of ∼*Introduction* is illicit when it would lead to a failed attempt to accept/assert a failed attempt to make a statement.

Similarly, we can reproduce our earlier argument as follows:

*x*
⎵∼Statement (a) is true.
————————————————
⎵It is true that ∼Statement (a) is true. *x* ⎵∼Statement (a) is true.
————————————————     ————————————————
⎵Statement (a) is true.     ¬Statement (a) is true.
————————————————————————————————
⊣∼Statement (a) is true.

We correctly deny both the Liar and the statement which the Liar negates.

It may be of some interest, and give a better understanding of the present proposal, if we extend our treatment to accommodate a 'chain' version of the Liar Paradox:

(c) Statement (d) is true.
(d) ∼Statement (c) is true.

If we suppose that statement (c) is true, we are led to reject this:

*x*
⎵Statement (c) is true.
———————————————— Paraphrase
⎵It is true that statement (d) is true.
———————————————— 'It is true that' Elimination
⎵Statement (d) is true.
———————————————— Paraphrase
⎵It is true that ∼Statement (c) is true.
———————————————— 'It is true that' Elimination
⎵∼Statement (c) is true.
———————————— ∼Elimination     *x*
¬Statement (c) is true.     ⎵Statement (c) is true.
———————————————————————————————— Negative Force Introduction
⊣Statement (c) is true.

And if we suppose-true (attempted) statement (c), we are led to reject that:

*x*
⎵Statement (d) is true.
———————————————— 'It is true that' Introduction
⎵It is true that statement (d) is true.
———————————————— Paraphrase
⎵Statement (c) is true.     ⊣Statement (c) is true.
———————————————————————————————— Negative Force Introduction
⊣Statement (d) is true.

We must reject the statement that (c) is true (which is different from either (c) or (d)), we must reject (c), and we must further reject (d):

$$
\dfrac{\dfrac{\dfrac{\overset{x}{\llcorner\sim\text{Statement (c) is true.}}}{\llcorner\text{It is true that }\sim\text{Statement (c) is true.}}\ \text{‘It is true that’ Introduction}}{\llcorner\text{Statement (d) is true.}}\ \text{Paraphrase} \qquad \dashv\text{Statement (d) is true.}}{\dashv\sim\text{Statement (c) is true.}}\ \text{Negative Force Introduction}
$$

We are committed to perform these denials:

$\dashv$Statement (c) is true.
$\dashv\sim$Statement (c) is true.
$\dashv$Statement (d) is true.

This shows us that attempted statement (d) (which is: $\sim$Statement (c) is true) is a failed attempt. However, if we suppose the negation of statement (c) as:

$\llcorner\sim$Statement (d) is true.

we are not led by similar reasoning (to that above) to reject this negation. Statement (c) is false, but it appears that we can safely and correctly accept the negation of this statement.

## 11. A logical system for paradoxical sentences

To gain a better understanding of these proposals for understanding denial and supposition, I will sketch a logical system for dealing with simple Liar sentences and attempted statements. This system is a model, in the model airplane sense, of an appropriate treatment of Liar sentences and attempted Liar statements. It does not provide a framework for some larger, more comprehensive logical theory that accommodates a variety of paradoxical sentences like those mentioned in section 7. But such a theory is not needed to achieve an understanding of the paradoxes.

In the artificial language of the theory to be developed here, there are some paradoxical sentences $\sim P$ such that $P$ is false but $\sim P$ is neither true nor false; in this language, both $P$ and $\sim P$ can correctly be denied. The paradoxical sentences in this language are intrinsically paradoxical, and all the paradoxical sentences are negations of (some) atomic sentences.

This system employs the artificial language $L_P$, which is quite similar to $L$. The plain atomic sentences of $L_P$ are divided into two non-empty classes, the *inoffensive* and the *troublemakers*. The troublemakers are atomic sentences $P$ for which $\sim P$ is paradoxical. We assume that we can tell which atomic sentences are troublemakers and which are not. We are treating troublemakers as a distinctive logical class of expressions; troublemakers contribute to the logical forms of sentences in which they occur. In $L_P$, the horseshoe ‘$\supset$’ is a primitive symbol.

Given the atomic sentences which are troublemakers, a sentence of $L_P$ is a *paradoxical* sentence iff it is the negation of a troublemaker. A sentence of $L_P$ is a *failed* sentence iff either it is a paradoxical sentence or it contains a paradoxical sentence.

An *interpreting function f for $L_P$* is a function defined on the atomic sentences of $L_P$ and the paradoxical sentences of $L_P$ which assigns F to each troublemaker atomic sentence, assigns either T or F to each inoffensive atomic sentence, and assigns * to a sentence $\sim P$ iff $P$ is a troublemaker. The value * is for speech acts that are failed attempts to make a statement (or for well-formed sentences that cannot be used to make a statement).

Given an interpreting function $f$ of $L_P$, the *valuation determined by f* is as follows:

(1) If $A$ is an atomic sentence or a paradoxical sentence, then its value is $f(A)$.
(2) If $A$ is not a troublemaker, then $\sim A$ has value T iff $A$ has value F, $\sim A$ has value F iff $A$ has value T, and $\sim A$ has value * iff $A$ has value *.
(3) '$[A \vee B]$' has value T iff both disjuncts have truth values (i.e. either T or F) and one disjunct has value T; '$[A \vee B]$' has value F iff both disjuncts have the value F; '$[A \vee B]$' has value * otherwise.
(4) '$[A \& B]$' has value T iff both $A$, $B$ have value T; '$[A \& B]$' has value F iff both conjuncts have truth values, and one conjunct has value F; '$[A \& B]$' has value *otherwise.
(5) '$[A \supset B]$' has value T if both components have truth values and either $A$ has value F or $B$ has value T; '$[A \supset B]$' has value F if $A$ has value T and $B$ has value F; '$[A \supset B]$' has value * otherwise.

The valuation is obtained by modifying one of the two Kleene matrices (See table 2).

A compound sentence is 'spoiled' by a spoiled component, for the connectives 'presume' that they are connecting genuine statements (sentences that represent genuine statements). Not all sentences with value * are paradoxical, but each such sentence has a paradoxical component.

Although a failed sentence cannot be used to make a statement, we will still consider reasoning that employs such sentences. If $A$ is a failed sentence, we regard it as representing a failed attempt to make a statement. Such a failed attempt cannot properly be accepted. However, we can imagine someone who, by mistake, thinks the act is a statement, and tries to accept it as being the case. Someone who makes such a

Table 2. Matrices for $L_P$.

| $A$ | $B$ | $\sim A$ | $[A \vee B]$ | $[A \& B]$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | T | F |
| T | * | F | * | * |
| F | T | T | T | F |
| F | F | T | F | F |
| F | * | T | * | * |
| * | T | * | * | * |
| * | F | * | * | * |
| * | * | * | * | * |

mistake might then reason correctly on the basis of their alleged assertion—our deductive system will 'sanction' such arguments.

It is a mistake to try to accept a failed attempt to make a statement, but any one can properly suppose a failed attempt at a genuine statement to be a true statement, and also properly suppose it to be a false statement. And it is correct to reject failed attempts at statements. If we have a pair of sentences $P$, $\sim P$ such that $P$ is a troublemaker and $\sim P$ is paradoxical, the illocutionary sequences '$\vdash P \rightarrow \vdash \sim P$', '$\vdash \sim P \rightarrow \vdash P$', '$\sqcup P \rightarrow \sqcup \sim P$', '$\sqcup \sim P \rightarrow \sqcup P$' are logically connected. It must be possible in the deductive system to establish each of these sequences as a theorem—and in deriving (say) '$\vdash \sim P$' from '$\vdash P$', the rules for negation and negative illocutionary force need not be employed.

All the rules, except *v Introduction* and $\sim$*Introduction*, of $S$ are also rules of $S_P$. The rule *v Introduction* now incorporates a restriction:

$$\frac{\vdash/\sqcup A}{\vdash/\sqcup[A \text{ v } B]} \qquad \frac{\vdash/\sqcup A}{\vdash/\sqcup[B \text{ v } A]} \qquad B \text{ does not contain a paradoxical component}$$

The rule $\sim$*Introduction* is dropped.

In addition, we need the following rules:

*Paradoxical Interchange*

$$\frac{\vdash/\sqcup P}{\vdash/\sqcup \sim P} \qquad \frac{\vdash/\sqcup \sim P}{\vdash/\sqcup P} \qquad P \text{ is a troublemaker}$$

This rule takes the place of the analysis of atomic sentences (statements) which cannot be carried out in a language of propositional logic. The rule gives the inferences that would be sanctioned by an analysis of troublemaker atomic sentences. Incorporating this rule in our system reflects our assumption that we can tell which atomic sentences are troublemakers and which are not.

*Modus Ponens*

$$\frac{\vdash/\sqcup A \quad \vdash/\sqcup[A \supset B]}{\vdash/\sqcup B}$$

$\supset$ *Introduction*

$$\frac{\begin{array}{c}\{\sqcup A\}\\ \\ \sqcup B\end{array}}{\vdash/\sqcup[A \supset B]}$$

The conclusion is an assertion if the only supposition in the subproof leading to $\sqcup B$ is the one in braces; otherwise, it is a supposition. Neither $A$ nor $B$ contains a paradoxical component.

*Excluded Middle Introduction*

$\vdash[A \text{ v } \sim A]$ $\quad A$ is not a troublemaker and does not contain a paradoxical component.

The legitimate applications of the principle $\sim$*Introduction*:

$$\frac{\dashv/\neg A}{\vdash/\sqcup \sim A} \qquad \sim A \text{ has a truth value (one of T, F)}$$

can be obtained as follows:

$$\frac{\dashv/\neg A \quad \vdash[A \ v \ \sim A]}{\vdash/\llcorner \sim A} \qquad EMI$$

for this form of disjunctive syllogism is a derived rule of $S_P$.

*Paradoxical Rejection*

$\dashv A$ $\quad$ $A$ contains a paradoxical component.

In the language $L_P$, if $P$ is a troublemaker, then $\sim P$ is a paradoxical sentence, but '$[P \ \& \ P]$,' '$[P \ v \ P]$,' '$[P \supset P]$' are neither troublemakers nor paradoxical. If $P$ is a troublemaker, then '$[P \supset P]$' is true, as are both '$\sim[P \ \& \ P]$' and '$\sim[P \ v \ P]$.' However, in $L_P$, a sentence '$[A \supset B]$' will not in every case have the same truth value as '$[\sim A \ v \ B]$.' If $P$ is a troublemaker, then '$[\sim P \ v \ P]$' has value *, although '$[P \supset P]$' is true. DeMorgan's Laws are also casualties. For troublemaker $P$, both '$[\sim P \ \& \ \sim P]$' and '$[\sim P \ v \ \sim P]$' have value *, even though '$\sim[P \ \& \ P]$' and '$\sim[P \ v \ P]$' are true.

A *commitment valuation* for $L_P$ is a function which assigns + to (some) completed sentences of $L_P$. A commitment valuation $\mathscr{E}$ of $L_P$ is *based on* an interpreting function $f$ iff (i) if $\mathscr{E}(\vdash A) = +$, then $f(A) = \mathrm{T}$; and (ii) if $\mathscr{E}(\dashv A) = +$, then $f(A) = \mathrm{F}$ or $f(A) = *$. A commitment valuation of $L_P$ is *coherent* iff it is based on an interpreting function of $L_P$.

Let $\mathscr{E}_0$ be a coherent commitment valuation of $L_P$. The *commitment valuation determined by* $\mathscr{E}_0$ is the function $\mathscr{E}$ such that (i) $\mathscr{E}(\vdash A) = +$ iff $A$ has value T for every interpreting function on which $\mathscr{E}_0$ is based; and (ii) $\mathscr{E}(\dashv A) = +$ iff $A$ has value F or value * for every interpreting function on which $\mathscr{E}_0$ is based.

We can see that if $\mathscr{E}_0$ is a coherent commitment valuation of $L_P$ and $\mathscr{E}$ is the commitment valuation that it determines, then if $P$ is a troublemaker, we have $\mathscr{E}(\dashv P) = \mathscr{E}(\dashv \sim P) = +$.

We need to slightly amend our definition of what it is that allows interpreting functions to satisfy completed sentences. An interpreting function $f$ *satisfies* a completed sentence $\llcorner A$ iff $f(A) = \mathrm{t}$ and *satisfies* a completed sentence $\neg A$ iff either $f(A) = \mathrm{t}$ or $f(A) = *$.

The definition of satisfaction for commitment valuations is unchanged: if $\mathscr{E}_0$ is a coherent commitment valuation, and $\mathscr{E}$ is the commitment valuation which it determines, then $\mathscr{E}_0$ satisfies an assertion or denial $A$ iff $\mathscr{E}(A) = +$.

What it is for a set of completed sentences to *logically require* an assertion, denial, or supposition is the same for $L_P$ as for $L$. It is a straightforward matter to establish that $S_P$ is sound and complete.

Our sketch of a formal-language account of simple paradoxical sentences shows that we can coherently deny what we cannot successfully negate. We can legitimately formulate Liar sentences, and coherently explore the deductive consequences of actually accepting/asserting attempted Liar Statements, as well as of denying/rejecting these attempted statements, and of supposing them to be either true or false. A really adequate language is one in which we can make statements about that very language. But such a language cannot contain an operator (operation) that invariably makes a true statement out of a false one. This is both a conceptual and an ontic limitation.

It is possible to view paradoxical sentences as pathological curiosities made possible by the expressive power of our language. Negation simply fails to 'keep up' with denial, for a denial: $\dashv A$, does not in every case amount to the same thing as the assertion of the corresponding negation: $\vdash \sim A$. We can deny/reject both an attempted paradoxical statement and the statement which the paradoxical statement would negate. For many logical purposes, it is convenient to employ logical theories in which the paradoxes are 'ruled out', either by employing an artificial language which does not allow the formulation of a paradoxical sentence, or by dealing with a limited semantic account which assigns no values to either troublemakers or paradoxical sentences. In such a theory, the negation operator can turn every true sentence into a false sentence, and every false sentence into a true sentence.

However, by restricting one's attention to logical theories in which paradoxical sentences are ruled out, one fails to come to grips with the sources of paradox, and gives up on understanding and capturing the expressive power of the languages we actually speak. With the understanding that we have achieved of denial and negation, and of our ability to formulate paradoxical sentences, we can see that there is to reason to think (or fear) that our ordinary language and our ordinary linguistic practices are doomed to inconsistency or incoherence. We can produce Liar sentences, but we cannot make Liar statements. There are no Liar statements.

## 12. Notes for a more comprehensive account

It is an interesting task to develop a full-scale comprehensive logical theory for dealing with Liar sentences, although such a theory is not needed for understanding the Liar Paradox. Such a theory will allow for a variety of paradoxical sentences: intrinsically paradoxical negations of atomic sentences, even intrinsically paradoxical atomic sentences, compound intrinsically paradoxical sentences, contingently paradoxical sentences, paradoxical chains of sentences, etc. We shall briefly consider some features of a comprehensive theory, without attempting to come up with the theory.

Let us say that a *fully adequate* logical theory is one for which the artificial language and semantic account 'capture' the expressive power of a natural language. The logical theory provides the resources for making, or representing, statements about both extra-linguistic objects and expressions of the language (or about the language acts represented by these expressions). The language contains a logical predicate for characterizing a sentence as true, as well as an '*It is true that*' operator. A fully adequate theory contains logical predicates for characterizing expressions syntactically and semantically; it contains sentential operators which are counterparts to these predicates (as '*It is true that*' is a counterpart to '*is true*'). The semantic account allows for sentences which are either true or false, as well as for paradoxical sentences which fail to represent statements. The deductive system of a fully adequate logical theory will enable us to infer the denial of paradoxical negative sentences and of the sentences which these negate. A *minimal* condition of adequacy is that the language and semantic account can accommodate paradoxical sentences, and the deductive system permits deductions/proofs whose conclusions are the denials of both troublemaker sentences and their paradoxical negations.

A *restricted* logical theory is one that doesn't 'aspire' to adequacy. In a restricted logical theory, there may or may not be sentences which are neither true nor false, but the negation of a false sentence is always a true sentence (and the negation of a true sentence is false). A restricted logical theory does not provide a place for paradoxical sentences. A satisfactory formal treatment of the Liar Paradox does not require a fully adequate interpreted theory, but the theory must be at least minimally adequate—in contrast to standard theories, which are restricted theories.

Devising a suitable formal language itself is not so difficult; some standard formulation will suffice. Providing an adequate semantic account is more difficult, though it is surely possible. (Doing so represents a merely technical problem.) Our logical theory for the language $L_P$ is an inaccurate or unrealistic model of a full-scale treatment in several respects. The most obvious shortcoming is that the language $L_P$ fails to accommodate the great variety of paradoxical sentences. A more serious shortcoming is that the theory assumes that it is possible to tell which sentences are paradoxical and which are not (because only the negations of troublemakers are paradoxical, and it is assumed that we can tell which sentences are troublemakers). However, there is no effective procedure for determining if a sentence (statement) is paradoxical or not—there is no such procedure, even for intrinsically paradoxical sentences.

In devising a semantic account for the artificial language, we need to be guided by the facts of our actual linguistic practice. Which statements are true or false is objectively determined, as is the matter of which sentential acts fail to be statements. Once the appropriate values are assigned to non-logical expressions of the artificial language, the semantic account must award truth and falsity to the right sentences, and must award * to paradoxical sentences and to sentences with paradoxical components. The account must also deal with puzzling but non-paradoxical sentences like this 'truth-teller':

(m) Statement (m) is true.

This does not occasion difficulties like those attending Liar sentences, for there is no property which a statement (m) would possess just in case it lacked that property. Since the sentence is not paradoxical, it should be possible to use the sentence to make a statement (it should be possible to make a statement represented by the sentence). My intuition is that (m) is not a true statement, because a true statement needs something to be true of. So statement (m) is false, and this is true:

$\sim$Statement (m) is true

Similarly, my intuition is that this sentence:

(n) [Statement (n) is true *v* $\sim$Statement (n) is true]

cannot be used to make a true statement, because one cannot make a statement true by calling it true. But then the first disjunct of (n) is false, and the second is paradoxical. Which means that (n) is not a statement after all but only an attempted statement. A systematic semantic account needs to award semantically appropriate (correct) values to sentences (m), (n), and other puzzling cases that can be formulated.

As customarily understood, the logical form of a sentence in an artificial logical language is a visible feature of that sentence and is determined by the distinctively logical expressions in the sentence, together with the syntactic structure of the sentence. It can be effectively determined whether or not two sentences of the language share a logical form. (I favour the conception of form according to which a sentence has more than one logical form, with the different forms differing in being more and less detailed.) A predicate for truth in an artificial language can reasonably be regarded as a distinctively logical expression, as can an '*It is true that*' operator for sentences. But in our discussion of the Liar Paradox and our formulations of Liar sentences, that a sentence is paradoxical is not determined by its logical form.

Which expressions are counted as logical, and what features are counted as belonging to logical form, are determined by the semantic treatment of the artificial language. A logical expression must 'come out the same' for all interpretations of the language. This can be accomplished either by specifying the value to be assigned to a logical expression, as is commonly done in interpreting the predicate for identity, or by assigning values to non-logical expressions, and then capturing the meaning of a logical expression by defining the valuation determined by a particular interpretation, as is commonly done with connectives and quantifiers.

We cannot make it a matter of logical form, for every paradoxical sentence, that it is paradoxical. For contingent Liar sentences depend for their paradoxical character on the truth or falsity of non-problematic sentences (statements). Even intrinsic Liar sentences depend for their paradoxical character on the values assigned to constants in the language. A sentence will be an intrinsic Liar sentence given one assignment, and non-paradoxical given some other. We might enlarge our conception of logical form by fixing it that a specified subclass of individual constants name the expressions in the language, and always name the same expressions. Having done this, their logical forms would determine that intrinsically paradoxical sentences have this character. However, I can see no compelling reason to reconceive logical form in this way.

The deductive system for a logical theory attempts to capture commitments linking illocutionary acts represented by completed sentences of the artificial language. But it does not aspire to capture all of those commitments. For example, some assertions deductively require others on the basis of considerations other than logical form. In a standard semantic account for an artificial language, non-logical predicates are not assigned meanings; they are assigned values. Different interpreting functions will assign different values to a non-logical predicate. Entailments which depend on the meanings of non-logical expressions will not 'show up' in such a semantic account and will not be captured by the deductive system.

A fully adequate logical theory allows us to make syntactic and semantic statements about expressions in the language, and the language acts which these expressions represent. But the logical theory does not itself give a particular assignment of values to constants and non-logical predicates in the language. Once we are given a particular interpretation which provides names for expressions in the language, we can speak of a *fully adequate interpreted theory*. This is a fully adequate logical theory which is supplemented with resources which make possible inferences employing (exemplifying?) principles of paraphrase. (These are principles like that authorizing us to infer '*It is true that M*' from '$\mu$ *is true*' when $\mu$ names $M$.) These principles may be adopted as additional rules of the deductive system or may be derived from assertions and denials which are given an axiomatic status.

A comprehensive theory for dealing with the Liar Paradox will be an at least minimally adequate interpreted theory. The interpreting function $f$ will determine which sentences are paradoxical. The deductive system $S^*$ which captures the commitment based on logical form in our artificial language $L^*$ will be supplemented with principles of paraphrase $\Pi$ which depend on $f$. This allows us to establish that sentences are paradoxical. For if we can establish both $\dashv A$ and $\dashv\sim A$, then either $\sim A$ is paradoxical or it has paradoxical components. The supplemented deductive system will not allow us to establish that sentences can be used to make (represent) statements. However, if sentence $B$ is a component of $A$, an assertion $\vdash A$ will commit a person to taking $B$ to be true or false. (A person can claim that $B$ is either true or false by making the assertion $\vdash[B \vee \sim B]$.) We simply recognize that many sentences are not paradoxical, or that many attempted statements are successful attempts. Our reasoning with them is not impeded by restrictions that prevent us from inferring $\vdash\sim A$ from $\dashv A$ when $\sim A$ is paradoxical or contains a paradoxical component.

The Liar Paradox is a puzzle, but it is not a paradox of a kind where correct reasoning from known premisses leads a person to accept false or contradictory statements. This puzzle gives us no reason to revise our understanding of truth, or to revise our opinion that contradictory statements ought only to be rejected, never accepted. We can correctly reason from '$\vdash$Statement (a) is true' to '$\vdash\sim$Statement (a) is true,' and we can correctly reason in the opposite direction, when statement (a) is: $\sim$Statement (a) is true. However, we *cannot* correctly assert either 'Statement (a) is true' or '$\sim$Statement (a) is true.' We *can* correctly deny both of these statements:

$\dashv$Statement (a) is true, $\dashv\sim$Statement (a) is true

Eklund is right to this extent: there *is* reasoning which is correct when we know our attempted statements to be genuine statements, which reasoning fails to be correct otherwise. We can infer '$\vdash\sim A$' from '$\dashv A$,' and '$\llcorner\sim A$' from '$\neg A$' only if $\sim A$ is known to be a genuine statement. This is the move that will get us into trouble if we make it indiscriminately. This is the move that is routinely 'underestimated', especially by those who claim that to deny a statement is merely to assert that statement's negation. Eklund seems to think that we are doomed to reason incorrectly when Liar sentences turn up, but there is no basis for such pessimism.

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